A 1D Lagrangian Adaptive Mesh Refinement (AMR) Algorithm

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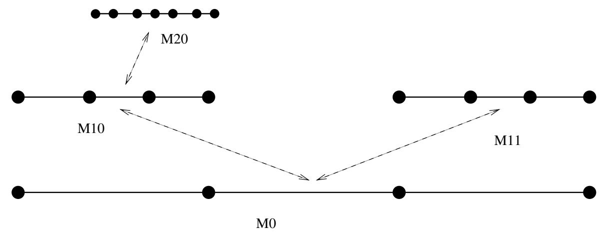
For the solution of fluid dynamics equations, the Adaptive Mesh Refinement (AMR) method promises to obtain solutions with less computational work than classical schemes. The AMR method is based on a modification of the mesh structure so that the computational mesh is n times denser in regions of interest, for example, near shock waves or contact discontinuities. In Lagrangian simulations, the AMR method also allows us to treat problems with rapid decreases of density in particular regions.

Our objective is to implement refinement and coarsening operators that can be plugged into the existing 1D ALE code [1]. After observing the inherent difficulties of the Lagrangian scheme [2] in the presence of mesh non-uniformities, especially near the transition of the shock wave from a denser mesh to a finer mesh and back [4], [3], the approach of [5] introduced in Eulerian coordinates, or of the following work in Lagrangian coordinates [6] was chosen.

We developed a 1D Lagrangian AMR space and time multi-mesh code. In this code, each computational mesh can have an arbitrary number of descendant meshes that are of n times higher resolution, where n is an odd number.

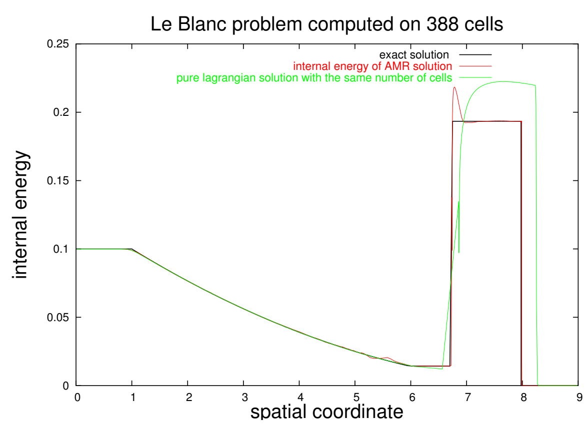
The Lagrangian solver computes the solution on each mesh from the coarsest to the finest, and the solution obtained on a given mesh is used as a boundary condition for all descendants of the mesh. We developed a C++ code to provide tools



Typical mesh data structure of the AMR solver, with 2 levels of refinement.

for multi-mesh computation, which will be easy to use in further research.

We also designed an error estimator, in the form of an L_2 norm of the difference between the solution and the piecewise constant reconstruction, and observed good performance in our tests. Our refinement strategy was developed using the method of beams, with satisfying results.



Internal energy of the LeBlanc problem [7] at time $t = 6$. We compare the AMR solution (red line) and the classical Lagrange solver solution (green line) computed with the same number of cells (388 cells). The black line shows the exact solution. We observe a significant improvement using AMR techniques.

We did observe difficulties with the shock injection on the finer mesh, resulting in unphysical discrepancies. This effect can be seen when an already developed shock is refined onto the new mesh, which happens when a number of refinement levels bigger than 4 are allowed. We identified the term of artificial viscosity [8] as the source of the problem, because artificial viscosity has non-physical, but cell number logical scale.

Based on the results in [8] and [9], we proposed a solution in the form of a non-conservative shock reconstruction on the finer mesh. We are further developing this method in order to overcome the conservation problem.

We plan to first develop the shock reconstruction method, and a refinement using even ratios of mesh resolution, in one dimension. We will then develop an extension of the algorithm to higher dimensions and to general polygonal meshes.
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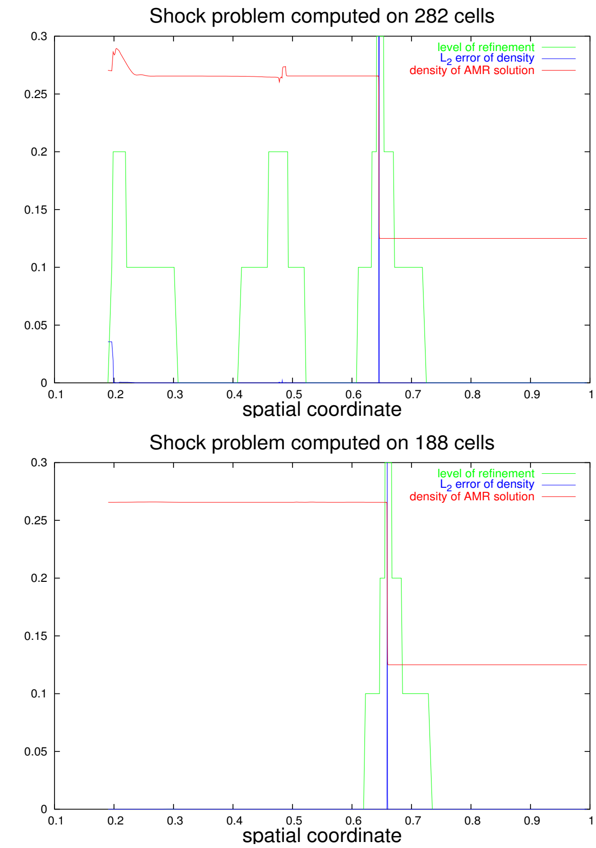

Shock reconstruction method on a steady-state shock. Upper figure shows the results when the shock is injected into the finer mesh without any reconstruction. Lower figure shows the results when the shock reconstruction is used.

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